Colouring diamond-free graphs

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The COLOURING problem is that of deciding, given a graph G and an integer k, whether G admits a (proper) k-colouring. For all graphs H up to five vertices, we classify the computational complexity of COLOURING for (diamond, H)-free graphs. Our proof is based on combining known results together with proving that the clique-width is bounded for (diamond, P_1+2P_2)-free graphs. Our technique for handling this case is to reduce the graph under consideration to a k-partite graph that has a very specific decomposition. As a by-product of this general technique we are also able to prove boundedness of clique-width for four other new classes of (H_1, H_2) -free graphs. As such, our work also continues a recent systematic study into the (un)boundedness of clique-width of (H_1, H_2) -free graphs, and our five new classes of bounded clique-width reduce the number of open cases from 13 to 8 (see [1]).

References

[1] Dabrowski K.K., Dross, F., Paulusma, D., Colouring diamond-free graphs, Proceedings of the 15th Scandinavian Symposium and Workshops on Algorithm Theory, *Leibniz International Proceedings in Informatics* **53**, 16:1-16:14, 2016.