Geometric Classification

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A fundamental theorem of Hadwiger classifies all rigid-motion invariant and continuous functionals on convex bodies (that is, compact convex sets) in $\mathbb{R}^n$ that satisfy the inclusion-exclusion principle,

$$\mathrm{Z}(K) + \mathrm{Z}(L) = \mathrm{Z}(K \cup L) + \mathrm{Z}(K \cap L)$$

for convex bodies K, L such that $K \cup L$ is convex. Under weak additional assumptions, such a functional Z is a finitely additive measure and hence Hadwiger's theorem is a counterpart to the classification of Haar measures.

Hadwiger's theorem characterizes the most important functionals in Euclidean geometry, the $n+1$ intrinsic volumes, which include volume, surface area, and the Euler characteristic. In recent years, numerous further functions and operators defined on the space of convex bodies and more generally on function spaces were characterized by their properties.

An overview of these results will be given.