

Inverse Approach In Ordinary Differential Equations And Nambu Bracket

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In the theory of ordinary differential equations we can find two fundamental problems. The direct problem which consists in a broad sense in to find the solutions of a given ordinary differential equation, and the inverse problem. An inverse problem of ordinary differential equations is to find the more general differential system satisfying a set of given properties.

Probably the first inverse problem appeared in Celestial Mechanics, it was stated and solved by Newton (1687) in *Philosophie Naturalis Principia Mathematica*, and it concerns with the determination of the potential field of force that ensures the planetary motion in accordance to the observed properties, namely the Kepler's laws.

The first statement of the inverse problem as the problem of finding the more general differential system of first order satisfying a set of given properties was stated by Erugin [1] and developed in [2]).

The new approach of an inverse problem which we propose uses as an essential tool the Nambu bracket.

In the seventies Nambu in [4] proposed a new approach to the classical dynamics based on an N dimensional Nambu–Poisson manifold replacing the even dimensional Poisson manifold and on $N - 1$ Hamiltonian H_1, \dots, H_{N-1} instead of a single Hamiltonian H . In the canonical Hamiltonian formulation, the equation of motion (Hamilton equations) are defined via the Poisson bracket. In Nambu's formulation, the Poisson bracket is replaced by the Nambu bracket. Nambu had originally considered the case $N = 3$. Although the Nambu formalism is a generalization of the Hamiltonian formalism its real applications are not as rich as the applications of this last one. We deduce new properties of this bracket which plays a very important role in the proof of all the results of this work and in its applications [3]. We observe that the applications of the Nambu bracket which we will give in this communication are original and represent a new direction in order to develop the Nambu ideas.

References

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