On some spectral properties of the $\overline{\partial}$-Neumann operator.

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We consider the $\overline{\partial}$-Neumann operator

$$N : L^2_{(0,q)}(\Omega) \longrightarrow L^2_{(0,q)}(\Omega),$$

where $\Omega \subset \mathbb{C}^n$ is bounded pseudoconvex domain, and

$$N_\varphi : L^2_{(0,q)}(\Omega, e^{-\varphi}) \longrightarrow L^2_{(0,q)}(\Omega, e^{-\varphi}),$$

where $\Omega \subseteq \mathbb{C}^n$ is a pseudoconvex domain and φ is a plurisubharmonic weight function. N is the inverse to the complex Laplacian $\Box = \overline{\partial}\,\overline{\partial}^* + \overline{\partial}^*\,\overline{\partial}$.

In addition, we describe spectral properties of the complex Laplacian $\Box_{\varphi,q}$ on weighted spaces $L^2(\mathbb{C}^n, e^{-\varphi})$. In this connection it is important to know whether the Fock space

$$\mathcal{A}^2(\mathbb{C}^n, e^{-\varphi}) = \{f : \mathbb{C}^n \longrightarrow \mathbb{C} \text{ entire } : \int_{\mathbb{C}^n} |f|^2 e^{-\varphi}\, d\lambda < \infty\}$$

is infinite-dimensional, which depends on the behavior at infinity of the eigenvalues of the Levi matrix of the weight function φ.

We discuss necessary conditions for compactness of the corresponding $\overline{\partial}$-Neumann operator related to Schrödinger operators with magnetic field.