Derived equivalences induced by big tilting modules

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A well-known theorem due to Happel [7] and Cline, Parshall and Scott [3] states that if R is a ring, $T \in \mathrm{Mod}R$ is a classical tilting module (in the sense of Miyashita) and $S = \mathrm{End}_R(T)$, then there is a derived equivalence

$$\mathbf{R}\mathrm{Hom}_R(T,-)\colon \quad \mathrm{D}(\mathrm{Mod}R) \rightleftarrows \mathrm{D}(\mathrm{Mod}S) \quad : - \otimes^{\mathbf{L}}_S T.$$

More recently, Bazzoni and collaborators [1, 2] showed that if T is an infinitely generated tilting module, one obtains a localization rather than equivalence.

In this talk, I would like to offer a new perspective on this situation. If T is an infinitely generated tilting module, then S is naturally a topological ring and one can consider contramodules over S (= modules which admit certain infinite R-linear combinations). By restriction of the codomain of $\mathbf{R}\mathrm{Hom}_R(T,-)$ to S-contramodules (which follows the spirit of [4, 5, 6]), we recover the derived equivalence above.

This all fits into a more general framework of a correspondence between big tilting modules in Grothendieck categories and big cotilting contramodules.

References

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