Non-escaping endpoints of entire functions

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Let $f_a(z) = e^z + a, a < -1$. The Julia set of f_a consists of an uncountable union of disjoint curves going off to infinity (a Cantor bouquet). Following several interesting results on the endpoints of these curves, we consider the set of non-escaping endpoints, that is, the endpoints whose iterates do not tend to infinity. We show that the union of non-escaping endpoints with infinity is a totally separated set by finding continua that separate these endpoints from infinity. This is a complementary result to the very recent result of Alhabib and Rempe-Gillen that for the same family of functions the set of escaping endpoints together with infinity is connected. Moreover, we present other functions in the exponential family as well as a function that was first studied by Fatou which share the same property of the non-escaping endpoints. Finally, we show how we can use the continua we have constructed in order to show that the union of the Fatou set with the set of points that escape to infinity'as quickly as possible' has the structure of a 'spider's web'.