Complexity and Simplicity in the dynamics of Totally Transitive graph maps

Lluís Alsedà

(in collaboration with Liane Bordignon and Jorge Groisman)

Departament de Matemàtiques, Edifici Cc, Universitat Autònoma de Barcelona, 08193-Cerdanyola del Vallès, SPAIN

Transitivity, the existence of periodic points and positive topological entropy can be used to characterize complexity in dynamical systems. It is known that for graphs that are not trees, for every $\varepsilon > 0$, there exist (complicate) totally transitive map (then with cofinite set of periods) such that the topological entropy is smaller than ε (simplicity).

First we will show by means of examples that for any graph that is is not a tree the relatively simple maps (with small entropy) which are totally transitive (and hence robustly complicate) can be constructed so that the set of periods is also relatively simple. To numerically measure the complexity of the set of periods we introduce the notion of *boundary of cofiniteness* defined as the smallest positive integer n such that the set of periods contains $\{n, n+1, n+2, \dots\}$. Larger boundary of cofiniteness means simpler set of periods. With the help of the notion of boundary of cofiniteness we can state precisely what do we mean by extending the entropy simplicity result to the set of periods: *there exist relatively simple maps such that the boundary of cofiniteness is arbitrarily large (simplicity) which are totally transitive (and hence robustly complicate).*

Moreover, we will show tat that, for circle and sigma maps, the above statement is a theorem. This is a good example on how the lack of knowledge about the structure of the set of periods can be overcome with appropriate simple arguments.