Stability of the topological pressure for continuously differentiable interval maps

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Assume that $T : [0,1] \to [0,1]$ is a C^1-map satisfying that $\{c \in (0,1) : T'c = 0\}$ is finite. Fix an $N \in \mathbb{N}$ with $N \geq \operatorname{card}\bigl(\{c \in (0,1) : T'c = 0\}\bigr) + 1$. Denote the family of all C^1-maps $S : [0,1] \to [0,1]$, which are piecewise monotonic with at most N intervals of monotonicity by $\mathcal{M}_N$. Obviously the conditions on T imply that $T \in \mathcal{M}_N$. The set $\mathcal{M}_N$ is endowed with the C^1-topology, this means with respect to the norm $\|S\| := \max_{x\in[0,1]} |Sx| + \sup_{x\in[0,1]} |S'x|)$. Then the stability of certain dynamical invariants of T under small perturbations is investigated. Observe that it is essential to assume that the number of intervals of monotonicity of the perturbation is bounded by the previously fixed number N, but one this number can be chosen arbitrarily large.

One obtains that for every continuous function $f : [0,1] \to \mathbb{R}$ the topological pressure is upper semi-continuous at T, this means $\limsup_{\widetilde{T}\to T} p(\widetilde{T}, f) \leq p(T, f)$. If f satisfies that $p(T, f) > \lim_{n\to\infty} \frac{1}{n} \max_{x\in[0,1]} \sum_{j=0}^{n-1} f(T^j x)$ (which is satisfied if $p(T, f) > \max_{x\in[0,1]} f(x)$ holds), then the topological pressure is continuous at T. Hence the topological entropy is continuous at T. In general the topological pressure is not lower semi-continuous. This will be shown giving an example. Suppose that $h_{\mathrm{top}}(T) > 0$ and that T has a unique measure μ of maximal entropy (this means $h_\mu(T) = h_{\mathrm{top}}(T)$). Then there exists an open neighbourhood U of T in $\mathcal{M}_N$ (with respect to the C^1-topology), such that every $\widetilde{T} \in U$ has a unique measure $\mu_{\widetilde{T}}$ of maximal entropy. Moreover, $\lim_{\widetilde{T}\to T} \mu_{\widetilde{T}} = \mu$ in the weak star-topology.